DYNAMIC RESPONSE OF MACHINE FOUNDATION: A STUDY OF STRUCTURAL DYNAMICS ON SOIL CONDITIONS

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Abstract

Machine foundations are critical components in industrial infrastructures. It should be designed such that it is capable to withstand the vibrational load induced by the operating machine ensuring safety, stability and minimal vibration transmission to the adjoining structures. Despite Nepal transitions toward industrialization, research on machine foundations—particularly for industrial facilities where large machines operate continuously—has been largely neglected in the country. Numerous research works have been done in the past few decades on the machine foundation globally, however literatures addressing the effect of variation of foundation mass on response of machine foundation is very limited. This study presents a comprehensive analysis of the dynamic response of machine foundation system under different possible scenarios. The analysis begins with a machine-foundation modeling and soil-structure interaction. The key parameter that affects the response on machine foundation such as foundation mass, soil type, loading frequency and natural frequency was observed analytically. The output demonstrates that the soil stiffness and the effect of foundation weight has a significant influence in the amplitude response. Corresponding to the frequency ratio separation criteria, stiff soils demonstrate low amplitude response with increased weight demand and vice versa. With increase in stiffness of soil the demand for disturbing frequency also increases to meet the resonance criteria. The findings of this study might provide a good insight for the design and analysis of machine foundation of industrial infrastructure of Nepal in future days contributing to improved resilience and stability.

Keywords: Machine Foundation, Response, Dynamics, Vibration, Harmonic Excitation

1. Introduction

Study of the dynamic response of machine foundations serves as an idea for determining the response of machine foundation system. Analysis of vibration of the system helps to ensure the stability and safety of the machinery within various industrial and power-plant structures. Machine foundation plays an important role in supporting heavy equipment such as turbines, generators, compressors and effectively transferring loads to the underlying soil.

If the vibrations and dynamic forces generated by the operating machine becomes excessive and uncontrollable, the machine may be forced to shut down, and even lead to fatigue failure of the machine components resulting in a catastrophic incident (G. Yung, 2014). So, it becomes important to understand the vibrational behaviour of the machine foundation.

Moreover, a proper design of machine foundations can increase performance and lifespan of the foundation and machineries. By avoiding resonance and ensuring stability, we can increase the lifespan of the structure. Machine foundations have specific requirements and guidelines to ensure compliance with safety and performance standards according to machine type. The IS- 2974 Code is widely recognized and accepted by many authors as a guideline for designing machine foundations. The basic goal in the design of a machine foundation is to limit its movement so that it neither endangers its operation nor disturbs people working in the immediate vicinity (Gazetas, 1983).

In the past, the calculations for this were simple and straightforward. An estimated dynamic factor was multi-

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plied by the static loads and the output was treated as the increased static load. However, in recent years, advances in computational tools, numerical modelling techniques, and experimental methods have facilitated more comprehensive analyses of machine foundation dynamics (Bhatia, 2008). Modern method analysis incorporates soil-structure interaction (SSI), considering the influence of soil dynamics on foundation behaviour. This has enabled engineers to accurately predict and evaluate factors such as resonance, natural frequencies, steady and transient responses under various loading frequencies and operating conditions of the machinery.

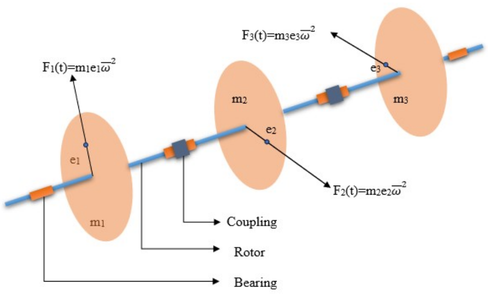

Figure 1. Coupled rotors inducing dynamic forces

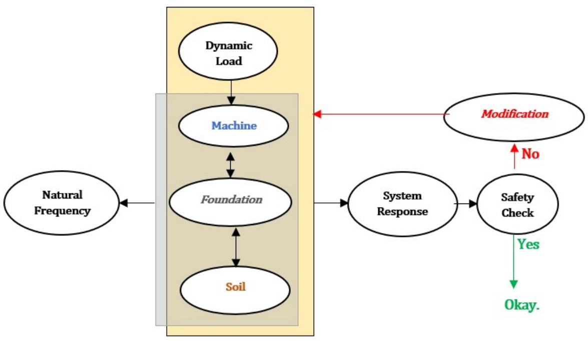

Figure 2. Machine-foundation system subjected to dynamic loads (modified from Bhatia, 2008)

Service factor, (S_f), is used to account for increased unbalanced force during the service life of the machine, generally greater than or equal to 2 (ACI, 2011).

Figure 1 illustrates when the mass centre of a rotating shaft does not coincide with the center of rotation, a dynamic force is produced. This force acts on the foundation-soil system, which vibrates in its own natural frequency. Figure 2 outlines the design methodology, where the analysis begins with the study of dynamic load acting on the system, which is transferred to the soil. The response of the system is influenced by the interaction of loading frequency and natural frequency. This influence generates a response that is checked in accordance with the established safety guidelines. If the response check satisfies the safety criteria, the design is accepted. However, in case the response does not satisfy the criteria, the foundation is modified to improve the results. So, designing a machine foundation is a complex process that requires multiple iterations to obtain an optimal solution.

Barkan (1962) introduced a simplified analytical approach for assessing the response of machine foundations by modelling them as a mass-spring-dashpot system. The study conceptualizes soil-structure interaction using an idealized linear spring element. Based on experiments, they established empirical correlations between the coefficient of subgrade reaction and the elastic and shear modulus of the soil. Additionally, the study proposed empirical relations to determine the spring constant of soil from subgrade reactions. It also emphasized the significance of frequency ratio separation criteria in preventing resonance and achieving optimal performance.

Gazetas (1983) provides a comprehensive review of analytical and numerical modelling approaches for machine foundations. The study explores dynamic impedance functions, key dimensionless parameters, and the effects of soil inhomogeneity, anisotropy, and non-linearity. They examined the response of two rigid massive foundations and offered practical guidelines for cost-effective predictions while highlighting challenges in field verification and nonlinear soil representation.

Shridhar (2021) provides a comparative response obtained using Lysmer's and Hall's analogs with Barkan's method. The study emphasized soil damping as a crucial factor in reducing vibration amplitudes and recommended design standards that incorporate soil vibration absorption rather than relying solely on frequency ratio separation criteria.

Bhandari and Sengupta (2014) analysed machine foundation response using the Linear Elastic Spring Method and the Elastic Half-Space Analog Method. The study concluded that the Elastic Half-Space Analog Method provided more accurate predictions than the IS code method. It also demonstrated that deeper embedment enhanced the stability by increasing natural frequency, thereby reducing the vibration.

G. Yung (2014) recommended conducting a forced vibration analysis for all critical machine foundations, considering dynamic soil or rock properties and machine-induced unbalanced forces as key design parameters. The study highlighted the significance of impedance functions in representing soil damping and stiffness. They suggested that assuming a rigid soil-foundation interface may underestimate the dynamic effects on flexible structures.

The novelty of the present work lies in its focused investigation of the effect of foundation mass on the dynamic response of machine foundations, a parameter that has been largely underexplored in existing studies. While several studies have addressed machine foundation dynamics; comprehensive analytical evaluations highlighting the combined influence of foundation mass, soil stiffness, and frequencies in determining amplitude responses are very limited. Furthermore, we try to addresses a critical gap in the vibrational engineering sector of Nepalese industrial infrastructure, fulfilling region-specific gap where such studies are lacking.

The outcome of this study will help the research community by providing a detailed combined understanding of how foundation mass, soil stiffness, and frequency interactions affect machine foundation behaviour, an area with limited research. For end users, it offers practical design insights to optimize foundation weight, avoid resonance, and enhance the safety and longevity of industrial infrastructure.

2. Modelling and Formulation

2.1. Dynamics of Structure

The vibration of the machine foundation is a forced vibration with harmonic excitation.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = P_o \sin(\varpi t) \tag{1}$$

Transient Response:

$$e^{-\xi\omega_0 t}\left[A\sin(\omega_D t) + B\cos(\omega_D t)\right] \tag{2}$$

Steady State Response:

$$C\sin(\varpi t) + D\cos(\varpi t) \tag{3}$$

The steady state response can be simplified as:

$$v(t) = \frac{P_o}{k} \cdot \frac{\sin(\varpi t)}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \tag{4}$$

A foundation block has six degrees of freedom (DOF) as shown in Figure 3: three along translation and three along rotation. However, in our study, horizontal and vertical vibration modes are prioritized.

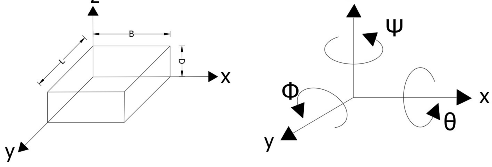

Figure 3. Dimensional axes and vibration modes

2.2. Dynamics of Soil

In the analysis and design of a machine foundation, the dynamics of the soil is as important as that of a structure and cannot be neglected. When a dynamic force is applied, the response of the foundation is highly influenced by the dynamic soil-structure interaction (SSI). The soil properties influencing structural response are:

- Energy transfer mechanism
- Soil mass participation
- Effect of foundation embedment
- Applicability of Hook's law to the soil
- Reduction in permissible soil stress
- Dynamic soil characteristics

There are several methods for analysis of soil dynamics for the vibration of machine foundations. They are listed as follows:

1. Empirical methods to evaluate the natural frequencies: Tschebotarioff (1951), Alpan (1961)
2. Methods based on elastic soil spring constants: Barkan (1962)
3. Methods based on linear massive elastic analog spring: Pauw (1954)
4. Methods based on considering the soil as an elastic half-space: Reissner (1936), Reissner and Sagoci (1944), Quinlan (1954), S. T. Yung (1954), Bycroft (1956), Hsieh (1962), Lysmer (1965), Lysmer and Richart Jr (1966), Hall (1967)
5. The impedance function method: Lysmer (1978), Novak et al. (1978), Roesset (1980)), Luco (1982), Gazetas (1983), Gazetas et al. (1985), and Dobry and Gazetas (1985)

In this study the method proposed by Barkan (1962) is utilized to study the dynamics of soil.

Coefficient of Subgrade Reaction

The coefficient of subgrade modulus is the ratio of the applied pressure to the induced deformation in the particular mode. There are 4 types of coefficients of subgrade reactions:

a. C_u, Coefficients of uniform compression

b. C_τ, Coefficients of uniform shear

c. C_θ, Coefficients of non-uniform compression

d. C_ψ, Coefficients of non-uniform shear

Barkan (1962) has suggested the relationship between the coefficient of subgrade reactions illustrated in Equations (5, 6, 7) and is accepted by majority of practitioners and also included in IS:5249 (1992). So, the Barkan's method is also known as Indian Standard method.

$$C_\tau = 0.5C_u \tag{5}$$

$$C_\theta = 2C_u \tag{6}$$

$$C_\psi = 1.5C_\tau \tag{7}$$

The coefficient of uniform compression (C_u) is the ratio of pressure to deformation in vertical mode and is determined by cyclic plate load test. However, Barkan (1962) has suggested the empirical correlation of (C_u), with elastic modulus (E), shear modulus (G), and Poisson's ratio (μ) as shown in Equations (8 and 10).

$$C_u = \frac{1.13E(1-\mu^2)}{A} \tag{8}$$

where, "A" is the area of base of foundation in contact with soil,

substituting,

$$G = \frac{E}{2(1+\mu)} \tag{9}$$

we get,

$$C_u = \frac{4Gr_o(1-\mu)}{A} \tag{10}$$

where r_o is the radius of an equivalent circular base.

The soil medium is then idealised as a linear spring element and the spring constant along the 6 DOFs can be defined as:

Stiffness about x-direction,	$K_x = C_\tau \cdot A$	(11)
Stiffness about y-direction,	$K_y = C_\tau \cdot A$	(12)
Stiffness about z-direction,	$K_z = C_u \cdot A$	(13)
Stiffness about θ-direction,	$K_\theta = C_\theta \cdot I_{xx}$	(14)
Stiffness about ϕ-direction,	$K_\phi = C_\phi \cdot I_{yy}$	(15)
Stiffness about ψ-direction,	$K_\psi = C_\psi \cdot I_{zz}$	(16)

2.3. Mechanics of Structure

The limiting frequency of the system can be determined once the spring stiffness is known. However, it should be noted that the actual dynamic response is significantly influenced by the natural frequencies of the system. The vibration response in the vertical mode (z-direction) and the torsional mode (ψ-direction) are purely uncoupled. In contrast, the remaining modes of vibration exhibit coupling effects, meaning that the translational frequencies are influenced by the rotational frequencies and vice versa.

$$N^2_{(x,\phi)} = \frac{1}{2\gamma_y}\left(L_x^2 + L_\phi^2\right) \pm \sqrt{\left(L_x^2 + L_\phi^2\right)^2 - 4\gamma_y L_x^2 L_\phi^2} \tag{17}$$

$$N^2_{(y,\theta)} = \frac{1}{2\gamma_x}\left(L_y^2 + L_\theta^2\right) \pm \sqrt{\left(L_y^2 + L_\theta^2\right)^2 - 4\gamma_x L_y^2 L_\theta^2} \tag{18}$$

Where $N(\omega_n)$ is the natural frequency, $L(\omega_L)$ is the limiting frequency, and γ_x, γ_y are the ratios of the mass moment of inertia about the centroid to the mass moment of inertia about the centre of gravity (CG) of the base of the machine foundation.

For uncoupled modes:

$$\omega_{n(z,\psi)} = \sqrt{\frac{K_{(z,\psi)}}{\text{Total mass}}} \tag{19}$$

For coupled modes: Combining the individual centroids in proportion to their masses, the overall centroid of the machine-foundation system should be determined. The eccentricity needs to be checked in accordance with IS:2974-I, 1982 (eccentricity $< 5\%$, in the horizontal plane).

$$x_0 = \frac{\sum(W_m \cdot x_m + W_f \cdot x_f)}{\sum(W_m + W_f)} \tag{20}$$

Mass moment of inertia of the machine and foundation about the CG of the base in contact with soil is defined as:

$$M_{(ox\_machine)} = \sum \frac{W_i}{g} \cdot \left(y_i^2 + z_i^2\right) \tag{21}$$

$$M_{(ox\_foundation)} = \sum \frac{m_i}{12} \cdot \left(y_i^2 + z_i^2\right) \tag{22}$$

Combined mass moment of inertia of machine and foundation about the c.g. of the foundation base plane:

$$M_{ox} = M_{ox\_machine} + M_{ox\_foundation} \tag{23}$$

Combined mass moment of inertia of machine and foundation about the overall centroid:

$$M_{mx} = M_{ox} - m \cdot \left(y_0^2 + z_0^2\right) \tag{24}$$

Similar is the explanation for the y and z components.

2.4. Finite Element Method

The Finite Element Method (FEM) is a computational technique used for the analysis of structures. It was developed in the mid-20th century and has now become an essential tool for tackling engineering challenges. Its versatility has made it a mainstay in various research areas, including in earthquake and structural engineering.

FEM divides the foundation and surrounding soil into smaller and manageable elements known as finite elements. By analysing each element on its own and then combining the results together, FEM makes it possible to solve problems that would be very difficult and largely time consuming to handle through traditional analytical methods. FEM offers a detailed understanding of the stresses, displacements, and potential failures in both the structure, foundation and the supporting soil.

One of the key advantages of FEM is its versatility. It can handle complex geometries, non-linear material properties, and various types of loading conditions. However, FEM is not without limitations. The accuracy of the results depends on factors such as mesh quality, element type, and the chosen mathematical model. The most important issue is the validation of result. Every package is a black box for the user and it has its associated limitations, some of which are explicit and some are implicit (Bhatia, 2008).

For the analysis of machine foundation vibration, modelling of the machine with rigid links or rigid beam elements and modelling foundation with solid element are considered good enough.

3. Machine, Foundation, and Soil

The selected machine parameters, as presented in Table 1, correspond to a real-life hydroelectric Turbo-Generator machine.

Table 1. Machine Properties (Troyer SpA)

Properties	Values
Rated Speed (rpm):	600
Weight of Turbine (kg):	9790
Weight of Stator (kg):	26000
Weight of Rotor (kg):	8420
Dynamic Force (kN):	9.50
Dynamic Torque (kN·m):	52.52
Load on DE Bearing (kN):	50
Load on NDE Bearing (kN):	17.50
Height of c.g. of Generator (mm):	900

Given that the machine foundations in hydropower plants feature its geometry with pockets, cutout, trenches, rise, and depression, exact modelling of such system is both challenging and time-consuming, especially when the study is analytical. So, a block model with a cutout for tailrace opening and rise for the non-drive machine was modelled without altering the machine footprints. The layout of the machine footprints along with the foundation configuration is illustrated in Figure 4.

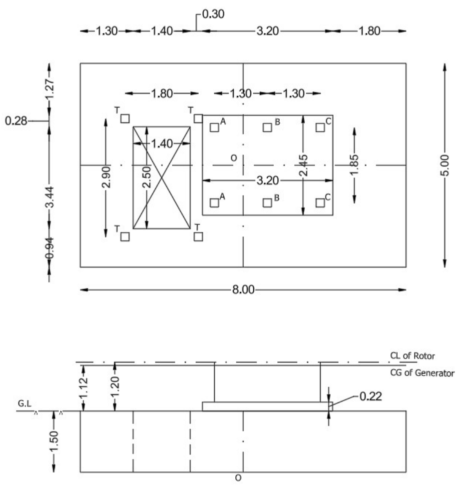

Figure 4. Foundation layout and machine footprints.

Table 2. Geotechnical data adopted

Soil Category	Cu (kg/cm^3) (IS 2974 Pt. I-1969)
Weak/Soft	2
Medium	5
Hard/Strong	7

In this study, the response of machine foundations across soft, medium, and hard soil types have been studied. The corresponding values of C_u for each soil types are presented in Table 2. The selection of variable parameters like foundation mass, soil type, loading frequency, and natural frequency has been made with clear consideration of their direct influence on the dynamic response and performance of machine foundations. Each parameter is chosen based on its critical role in determining amplitude response and resonance characteristics.

To consider natural frequency as a variable, we examined the interplay between soil stiffness and foundation weight. Multiple simulations were conducted for different foundation geometries by increasing the thickness of foundation across these soil conditions.

Similarly, when observing the response to varying excitation frequencies, the natural frequency was kept constant for a foundation depth of 1.5 m for soft, medium, and hard soil. This showcases that the variables of the study are soil type, foundation weight and parameters affecting the frequency ratio i.e. natural and loading frequencies.

4. Result and Interpretation

4.1. Effect of Natural Frequency

The output was drawn for horizontal and vertical amplitude of the foundation at DOF location for varying natural frequency and a constant rated excitation frequency as stated by the machine manufacturer. The natural frequency was varied by adjusting the foundation weight through changes in depth. Multiple analytical solutions were carried out across soft, medium, and hard soils, and the results were compared.

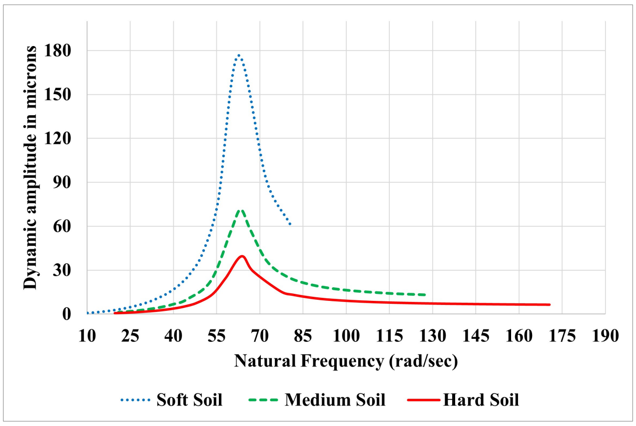

Figure 5. Horizontal response along natural frequency.

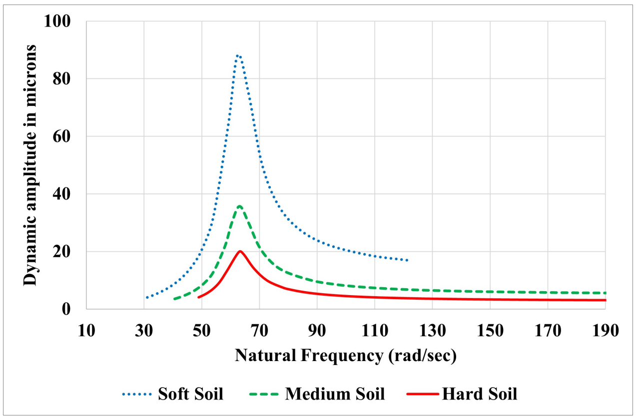

Figure 6. Vertical response along natural frequency.

It was observed that the soft soil exhibits the highest response across the natural frequency. Similarly, in both horizontal and vertical modes of vibration, the amplitude increases by approximately 1.8 times in medium soil and 4.4 times in soft soil when compared to in hard soil (Figure 5 and 6). Regardless, the soil medium, the resonance peak is observed at the same natural frequency. This is because the criteria for frequency ratio remains the same for all soil types at a constant excitation frequency.

4.2. Effect of Excitation Frequency

The response of the machine foundation at DOF location was also observed at different operating frequencies and the results are comparable to a similar previous study (Shridhar, 2021). It was observed that the soft soil amplifies vibration at lower frequencies making it more susceptible to dynamic excitation whereas, the hard soil reduces the vibration response significantly at the same exciting frequency. With increase in soil stiffness the harmonic load demand of foundation is observed to increase to meet the resonance. Here, despite the soil medium, the resonance peak exhibits the same magnitude of dynamic amplitude. It is observed that in horizontal mode, the resonant excitation frequency is 1.5 times higher in medium soil and 1.8 times higher in hard soil compared to in soft soil. Similarly, in vertical mode, the frequency increases by 1.6 times in medium soil and 1.2 times in hard soil (Figure 7 and 8).

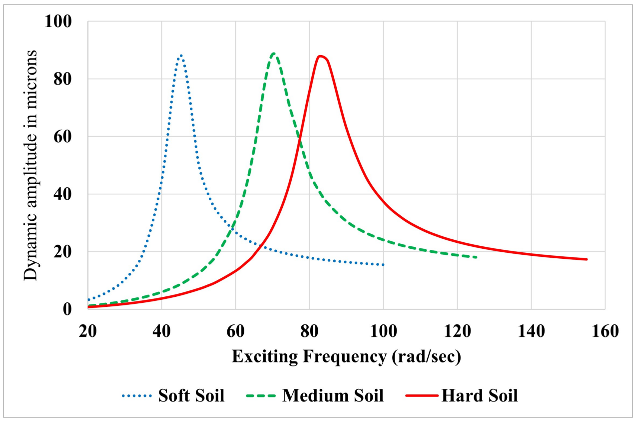

Figure 7. Horizontal response along exciting frequency.

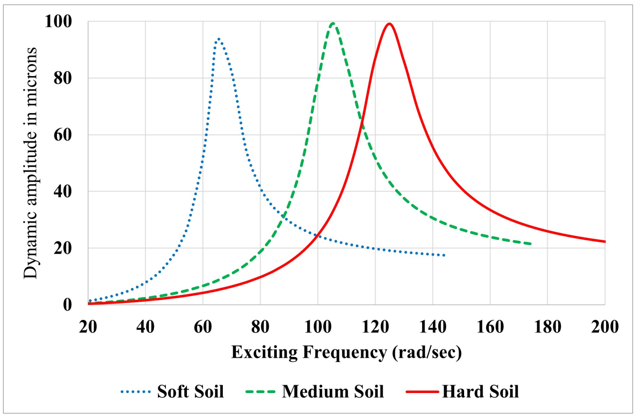

Figure 8. Vertical response along exciting frequency.

4.3. Effect of Foundation Mass

It is observed that foundations placed on soft soil exhibit higher amplitudes and support lighter weights for the same frequency. In contrast, hard soil with heavier foundations display minimal dynamic response. The performance in medium soil lies between these two extremes, providing a balance between dynamic response and structural demand. This observation highlights the importance of base isolation techniques to reduce structural demands, particularly on very stiff soils. The results of variation of response in horizontal and vertical mode of vibration due to effect of foundation weight are illustrated in Figures 9, 10, and 11.

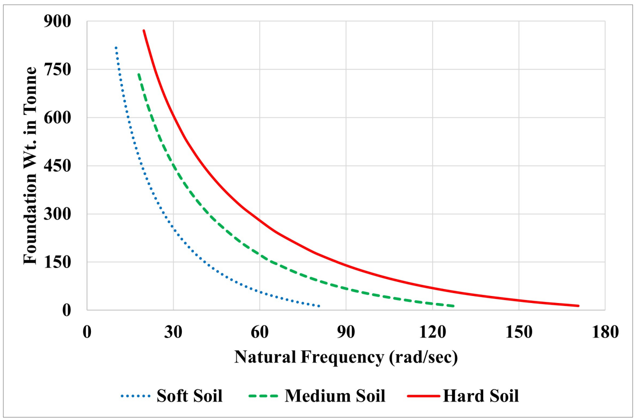

Figure 9. Variation frequency along foundation mass in horizontal mode

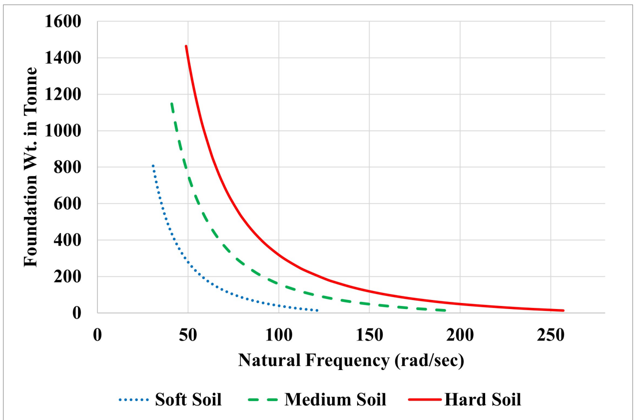

Figure 10. Variation frequency along foundation mass in vertical mode

Frequency ratio being under-tuned for low-speed rotary machine gradually gets closer to unity (1) when mass of the foundation get gradually increased. As frequency ratio is inversely proportional to the natural frequency, increase in foundation weight causes to decrease the natural frequency of the system consequently increasing the frequency ratio. Throughout this interplay, the response attains the maxima when the frequency ratio separation becomes equal and gradually decreases again on over-tuned state at post resonant condition.

As shown in Figure 11, a steep response is observed in

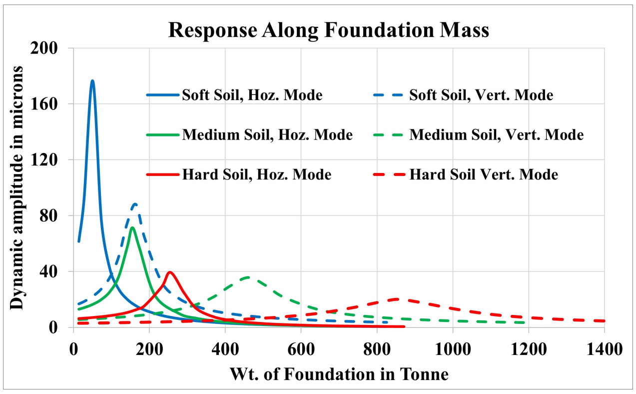

Figure 11. Dynamic response along foundation mass

the horizontal mode, indicating that foundation should be preferred in over-tuned state along horizontal mode. Slight change in operational or environmental condition might cause the response to reach resonance, in reverse for vertical mode; it is always preferable to design the low-speed rotary machine foundation at under-tuned state for cost effectiveness.

5. Conclusion

The study shows that in both the horizontal and vertical modes, the resonant amplitude along natural frequency variation increases by approximately 1.8 times in medium soil and 4.4 times in soft soil compared to in hard soil. In horizontal mode, the resonant excitation demand is 1.5 times higher in medium soil and 1.8 times higher in hard soil compared to in soft soil. Similarly, in vertical mode, the loading demand increases by 1.6 times in medium soil and 1.2 times in hard soil.

Based on the frequency ratio separation criteria, it was observed that the demand of the structure increases with increase in stiffness of the soil support. Hence, vibration isolation technique might be useful to increase the flexibility of the supporting system to reduce the structural demand.

Moreover, it might be effective to design the block foundation with over-tuned horizontal mode and under-tuned vertical mode in smaller foundation. The intersection of the response curves could provide valuable insights for weight calculation of foundation for vibration amplitude in both directions. However, for larger mat, the system may directly shift to an over-tuned state in both directions, consequently decreasing the amplitude by adding mass.

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